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考虑启动压力梯度的低渗透油藏不稳定渗流模型*

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摘要：考虑启动压力梯度的不稳定渗流数学模型解法十分繁琐，主要原因是控制方程具有强烈的非线性，同时流体流动边界并非恒定不变。为简化计算，利用油藏压力分布近似表达式，研究了低渗透储层不稳定渗流压力分布特征；进而根据物质平衡原理，利用牛顿迭代方法计算得到动边界在不同时刻的运动规律；最后，作为比较和验证，分别利用近似解法和解析解法研究了定产生产时的储层压力分布特征和井底压力变化规律。计算结果表明，启动压力梯度阻碍了地层能量的传播和流体的运移，消耗了部分地层能量，不利于油井生产。

关键词：低渗透；启动压力梯度；不稳定渗流；近似解；半解析解

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物理模拟实验研究结果表明，低渗透、特低渗透油藏存在着启动压力梯度[1]。流体在地层中流动时，不再符合达西定律。求解低渗透油藏不稳定渗流多采用数值解法[2]或者拉氏空间解析解[3]，但这两种解法的计算量都很大，同时拉氏空间解数值反演时部分曲线跳跃性很强。笔者提出了低渗透油藏不稳定渗流的近似解和半解析解，简化了计算。

1　低渗透油藏不稳定渗流近似解

由于低渗透油藏不稳定渗流动边界是时间的函数，它随着时间的推移逐渐增大。于是，任一时刻油藏在径向上可分为 2个区域：压力波影响到的激动区和没有影响到的未激动区，激动区的外边界可以认为是该时刻的供给边缘。

考虑启动压力梯度的低渗透油藏不稳定渗流方程为

$$\frac{1}{r}\left\{\frac{\partial}{\partial r}\left[r\left(\frac{\partial p}{\partial r}-G\right)\right]\right\}=\frac{1}{\eta}\frac{\partial p}{\partial t} \tag{1}$$

式中，r为半径，m；η为地层导压系数，m^2/s；G为启动压力梯度，Pa/m；p为压力，Pa；t为时间，s。

初始条件

$$p(r, 0)=p_e \tag{2}$$

边界条件

$$\begin{cases}\left(\frac{\partial p}{\partial r}-G\right)\Big|_{r=r_w}=\frac{Q(t)\mu}{2\pi khr_w}\\ \left(\frac{\partial p}{\partial r}-G\right)\Big|_{r=R(t)}=0\\ p=p_e \qquad r\geqslant R(t)\end{cases} \tag{3}$$

式中，r_w为井眼半径，m；k为渗透率，m^2；μ为流体黏度，Pa·s；$Q(t)$为产量，m^3/s；h为油层厚度，m；$R(t)$为动边界半径，m；p_e为原始地层压力，Pa。

假设激动区压力分布可以由坐标的对数和指数多项式表示[4]，根据精度要求，此处取其前 3项，即

$$p=a_0\ln\frac{r}{R(t)}+a_1+a_2\frac{r}{R(t)},\quad r_w\leqslant r\leqslant R(t) \tag{4}$$

上式对 r取导数，同时代入式（3）得到 3个方程，解出 a_0、a_1、a_2，可以得到

$$p=p_e+\frac{Q(t)\mu}{2\pi kh}\left[\ln\frac{r}{R(t)}+1-\frac{r}{R(t)}\right]-G[R(t)-r] \tag{5}$$

只要求出激动区外边界的移动规律，就可以得到不同时刻的地层压力分布特征。

1.1　油井以定产量生产

由物质平衡方程，单位时间内采出的液量等于

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同一时间间隔中地层激动区内液体弹性储量的改变量,即

$$Q = C_t \cdot \frac{\mathrm{d}}{\mathrm{d}t}[V(t)\,\overline{\Delta p}] \qquad (6)$$

其中
$$V(t) = \pi[R^2(t) - r_w^2]\phi h \qquad (7)$$

$$\overline{\Delta p} = p_e - \bar{p} \qquad (8)$$

式中, C_t为综合压缩系数, Pa^{-1}; ϕ为孔隙度,小数。

激动区内地层加权平均压力 $\bar{p}$可由下式求出。

$$\bar{p} = \frac{1}{\pi[R^2(t) - r_w^2]\phi h}\int_{r_w}^{R(t)} p(r, t)\,2\pi h r\phi\,\mathrm{d}r = p_e - \frac{Q\mu}{12\pi kh} - \frac{1}{3}R(t)G \qquad (9)$$

将式(7)~(9)代入式(6),并在区间[0, t]积分,可以得到

$$12\eta t = [R^2(t) - r_w^2]\left(1 + \frac{4\pi khR(t)G}{Q\mu}\right) \qquad (10)$$

可以用试算法或直线法对式(10)进行求解,得到不同时刻激动区外边界 $R(t)$的值。进而利用公式(5)求出任一时刻地层中的压力分布。

1.2 油井以变产量生产

对方程(1)在区间$[r_w, R(t)]$上积分,同时代入边界条件(3),有

$$-\frac{Q(t)}{2\pi h\phi C_t} = \int_{r_w}^{R(t)} r\frac{\partial p}{\partial t}\mathrm{d}r \qquad (11)$$

对上式在[0, t]范围内积分,得到

$$-\frac{1}{2\pi h\phi C_t}\int_0^t Q(t)\,\mathrm{d}t = \int_{r_w}^{R(t)}[\overline{p(t)} - \overline{p(0)}]\,r\mathrm{d}r = -\int_{r_w}^{R(t)}\left[\frac{Q(t)\mu}{12\pi kh} + \frac{1}{3}R(t)G\right]r\mathrm{d}r \qquad (12)$$

于是,

$$\int_0^t Q(t)\,\mathrm{d}t = [R^2(t) - r_w^2]\left[\frac{Q(t)}{12\eta} + \frac{1}{3}\pi\phi C_t hR(t)G\right] \qquad (13)$$

同样,也可利用试算法或直线法对式(13)求解得出不同时刻激动区外边界半径 $R(t)$,将其代入公式(5)可求出任一时刻地层中的压力分布。可以看出,式(10)是式(13)中的产量 $Q(t)$恒定的特殊形式。

2　低渗透油藏不稳定渗流半解析解

渗流控制方程、初始条件以及边界条件同(1)~(3)式。

令 $\Psi = p - G(r - r_w)$, (1)式可以变形为

$$\frac{\partial^2\Psi}{\partial r^2} + \frac{1}{r}\frac{\partial\Psi}{\partial r} = \frac{1}{\eta}\frac{\partial\Psi}{\partial t} \qquad (14)$$

引入中间函数 ξ,令 $\xi = r^2/(\eta t)$,上式可以化为

$$\frac{\partial^2\Psi}{\partial\xi^2} + \left(\frac{1}{4} + \frac{1}{\xi}\right)\frac{\partial\Psi}{\partial\xi} = 0 \qquad (15)$$

将 Ψ和 ξ代入方程(3),考虑边界条件

$$\xi\frac{\partial\Psi}{\partial\xi}\bigg|_{\xi = \frac{r_w^2}{\eta t}} = \frac{Q\mu B}{4\pi kh} \qquad (16)$$

则方程(15)的解为

$$\frac{\partial\Psi}{\partial\xi} = \frac{Q\mu B}{4\pi kh}\mathrm{e}^{\frac{r_w^2}{\eta t}} \cdot \frac{\mathrm{e}^{-\frac{\xi}{4}}}{\xi} \qquad (17)$$

又当 $r = R(t)$时

$$\Psi = p - G[R(t) - r_w],\ \xi = R^2(t)/(\eta t)$$

将其代入(17)式,并在$[r, R(t)]$区间积分,最终可以得到无穷大地层任一时刻地层压力分布

$$p = p_e - \frac{Q\mu B}{4\pi kh}\mathrm{e}^{\frac{r_w^2}{\eta t}}\left[-\mathrm{Ei}\left(-\frac{r^2}{4\eta t}\right) + \mathrm{Ei}\left(-\frac{R^2(t)}{4\eta t}\right)\right] - G[R(t) - r] \qquad (18)$$

2.1　油井以定产量生产

将 $-\mathrm{Ei}(-x)$展开,代入压力解析解,根据物质平衡方程可以确定动边界随时间的变化

$$Qt = \pi[R^2(t) - r_w^2]\phi hC_tY \qquad (19)$$

其中

$$Y = \frac{Q\mu B}{2\pi khR^2(t)}\int_{r_w}^{R(t)}\left[a_0 + a_1\frac{r^2}{4\eta t} + a_2\left(\frac{r^2}{4\eta t}\right)^2 + a_3\left(\frac{r^2}{4\eta t}\right)^3 + a_4\left(\frac{r^2}{4\eta t}\right)^4 + a_5\left(\frac{r^2}{4\eta t}\right)^5 + a_6\ln\left(\frac{r^2}{4\eta t}\right)\right]r\mathrm{d}r - \frac{Q\mu B}{2\pi kh}\left[a_0' + a_1'\frac{R^2(t)}{4\eta t} + a_2'\left(\frac{R^2(t)}{4\eta t}\right)^2 + a_3'\left(\frac{R^2(t)}{4\eta t}\right)^3 + a_4'\left(\frac{R^2(t)}{4\eta t}\right)^4 + a_5'\left(\frac{R^2(t)}{4\eta t}\right)^5 + a_6'\ln\left(\frac{R^2(t)}{4\eta t}\right)\right] + \frac{1}{3}R(t)G$$

式中, a_0 ~a_6 和 a_0' ~a_6'分别是与$\frac{r^2}{4\eta t}$和$\frac{R^2(t)}{4\eta t}$有关的系数。

2.2　油井以变产量生产

当井产量是以任意的一条曲线变化时,可用杜哈美原理求解这类带有时间变量边界条件的弹性不稳定渗流。

根据杜哈美原理,任一时刻地层压力分布为

$$p(r, t) = f_0(r) + \int_0^t\frac{\partial}{\partial t}F(r, t - \tau)\,\mathrm{d}\tau \qquad (20)$$

式中, $f_0(r)$为 $t=0$时在 D 域各处的解, $F(r, t - \tau)$为对应同类方程非时间变量边界条件的解。由方程(13)推导出无穷大地层定产生产数学模型的解为

$$p(r,t) \approx p_e - \frac{Q\mu B}{4\pi kh}\left[\int_{\frac{r^2}{4\eta(t-\tau)}}^{\frac{R^2(t)}{4\eta(t-\tau)}} \frac{e^{-y}}{y}\,dy\right] - \lambda[R(t) - r] = F(r, t-\tau) \quad (21)$$

对式(21)求导，将非时间变量的边界条件替换为时间边界变量条件，杜哈美积分可以写成

$$\int_0^t \frac{\partial}{\partial t}F(r, t-\tau)\,d\tau = -\frac{\mu B}{4\pi kh}\int_0^t \frac{Q(\tau)}{t-\tau}\cdot \left[e^{-\frac{r^2}{4\eta(t-\tau)}} - e^{-\frac{R^2(t)}{4\eta(t-\tau)}}\right]d\tau - \lambda[R(t) - r_w] \quad (22)$$

不同时刻地层压力分布为

$$p(r,t) = p_e - \frac{\mu B}{4\pi kh}\int_0^t \frac{Q(\tau)}{t-\tau}\cdot \left[e^{-\frac{r^2}{4\eta(t-\tau)}} - e^{-\frac{R^2(t)}{4\eta(t-\tau)}}\right]d\tau - \lambda[R(t) - r_w] \quad (23)$$

3 近似解与半解析解结果比较

均质无穷大地层中心一口井以定产量生产，地层参数如下：地层压力 $p_e = 20$ MPa，渗透率 $k = 3\times10^{-3}$ μm^2，孔隙度 $\phi = 0.2$，综合压缩系数 $C_t = 1.5\times10^{-3}$ MPa^{-1}，启动压力梯度 $G = 0.01$ MPa/m，流体黏度 $\mu = 3$ mPa·s，有效厚度 $h = 10$ m，井半径 $r_w = 0.1$ m，油井产量 $Q = 10$ m^3/d。

3.1 启动压力梯度对动边界扩展的影响

利用近似解和半解析解2种求解方法得到了启动压力梯度不同时油藏激动区动边界随时间向外拓展规律。随着时间的延长，动边界不断向外推移。同时，油藏的启动压力梯度越大，相同时间内激动区外边界半径越小，压力波及的范围越小，动边界运移的速度也越慢；在流动后期可以认为动边界扩展到一定程度后就不再继续扩展。比较近似解和半解析解的结果可以看出，近似解得到的激动区外边界半径移动速度要略大于半解析解的结果。

3.2 启动压力梯度对井底流压的影响

用近似解和半解析解得到了启动压力梯度对井底流压变化影响规律。随时间的延长，定产生产时井底流压不断降低，生产初期井底压力的降幅要远远大于生产中后期井底压力的变化，启动压力梯度越大，井底流压降得越快。

3.3 启动压力梯度对地层压力分布的影响

图1为定产生产30 d时不同的启动压力梯度对地层压力分布的影响。启动压力梯度越大，压力波及的范围就越小，近井地带地层压力下降速度也越快，地层压力越低。从图2中可以看出，随时间的延长，地层中的压降漏斗在径向上也不断扩展加深，但降低幅度逐渐减小。

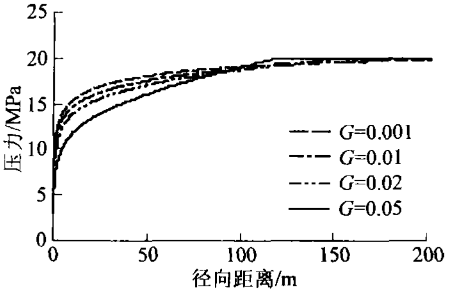

图1 启动压力梯度对地层压力分布影响

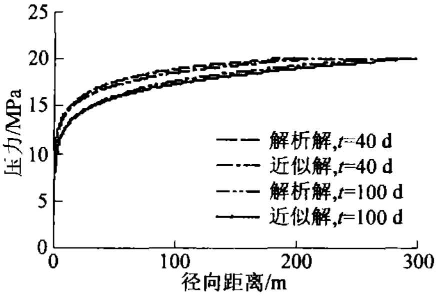

图2 地层压力分布比较(G=0.01 MPa/m)

以上分析表明，半解析解与近似解之间吻合较好；同时说明推导计算过程是可靠的。

4 结论

(1)考虑启动压力梯度的低速非达西渗流的近似解和半解析解，求解方法简便，满足工程计算要求，两者之间具有较好的一致性。

(2)低渗透油藏不稳定渗流时，启动压力梯度影响压力波的传播速度，也影响油水井的单井控制面积，因此，优化合理井排距时应加以考虑。

(3)启动压力梯度增大了地层流体的渗流阻力，降低了同一时期的井底流压和近井地带的地层压力，使地层压降漏斗变陡，增加了油藏开发难度。

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namics of the rod at the bottom and apprehend the working condition of the pump. The fluid pressure in the pump barrel was measured by a dynamics rod detecting instrument. The study showed that the fluid pressure in the pump barrel was varied periodically. The fluid pressure in the pump barrel was for different stroke, where the pressure in the lower stroke was larger than that of upper stroke. The study also showed that the pressure in the pump barrel was affected by the leakage of the valves, and the fluid flow resistance between rod and tubing was very large.

Key words: rod pump; pump-barrel; pressure; variation; regularity

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Design and appraisal of horizontal wells deep well lifting technology. CAO Gang, WU Xiao-dong, ZHU Xin-feng, LI Wei, ZHOU Han-peng, TANG Long. ODPT, 2006, 28(5): 52-54

Abstract: Sucker rod lift is one of the most popular artificial lift methods in oil industry. Currently, it has been widely applied in vertical wells as well as in horizontal wells. This paper presents several new types of sucker rod lifting technology for deep horizontal wells in low permeability reservoir, which include differential sucker rod pump and pump lifting technique for highly deviated wells. And the key techniques, such as the pump design in highly deviated well and the centralization of sucker rod, are introduced in detail. Indoor experiments and field application indicated that the differential sucker rod pumping and the new type of pumping technology in highly deviated wells had the characteristics of high pump efficiency and good reliability, which could meet the pumping requirements in deep horizontal wells in Daqing peripheral oilfield.

Key words: horizontal wells; sucker rod lift; deep well lifting; highly deviated pump; differential double-pump lifting

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Evaluating method for heterogeneity of multiple porosity sandstone. FAN Yu-ping, LIU Hui-qing. ODPT, 2006, 28(5): 55-57

Abstract: Heterogeneity fractal description of multiple porosity sandstone was established in this paper. Mercury injection experiment was conducted over 25 core samples, which could help to calculate the rock fractal factor and quantitative describe the heterogeneity of multiple porosity sandstone. Compared with coefficient of variation, the fractal factor had better interrelationship with rock permeability, porosity, threshold pressure, minimum wet saturation, and mean pore size. There was a linear relation between fractal factor and minimum wet saturation, which was suitable for both absolute homogeneous and heterogeneous rock. As the filling of mercury in the knaggy pore space made the pore flow channel more uniform, the fractal factor in the mercury ejection process is smaller than that of mercury injection process.

Key words: sandstone; pore structure; heterogeneity; mercury injection; fractal factor

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Unsteady seepage model for low permeability reservoir with threshold pressure gradient. HAO Fei, CHENG Lin-song, LI Chun-lan, CAO Gong-ze, HE You-an. ODPT, 2006, 28(5): 58-60

Abstract: The solution method of instability seepage flow with Threshold Pressure Gradient (TPG) consideration was extremely complex because the governing equation was non-linear and flow boundary was not constant. In order to simplify the solution calculation, pressure characteristic about instability seepage flow in low permeability reservoir was studied with the approximate equation of formation pressure. Furthermore, the movement rule of the flow boundary at different times was gotten with material balance approach and the Newton iteration calculation. Finally, the pressure characteristic of reservoir and well under the condition of invariable productivity was given with approximate solution and analytic solution separately. The study shows that the TPG resists the propagation of energy and migrate of fluid, consumes part of the formation energy, and is harmful to the well production.

Key words: low permeability; threshold pressure gradient; unsteady seepage; approximate solution; semi-analytical solution

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Pressure drop calculation of oil-water two phase variable mass stratified flow in horizontal pipelines. WEI Jian-guang, WANG Zhi-ming, WANG Xiao-qiu. ODPT, 2006, 28(5): 61-64

Abstract: This paper presents a model for pressure-drop calculation of oil-water two-phase variable mass stratified flow in horizontal pipelines. The influence of inflow through perforation channels was taken into consideration and the model of oil-water variable mass stratified flow in horizontal pipelines was established by infinitesimal analysis. The mass and momentum conservation equations were established for oil and water phase respectively in the model, where the influence of wall inflow on pressure drop was considered. The basic model and pressure drop calculation model were established for oil-water two-phase variable mass